Chapter 4 Understanding Interest Rates

PREVIEW

Interest rates are among the most closely watched variables in the economy. Their movements are reported almost daily by the news media, because they directly affect our everyday lives and have important consequences for the health of the economy. They affect personal decisions such as whether to consume or save, whether to buy a house, and whether to purchase bonds or put funds into a savings account. Interest rates also affect the economic decisions of businesses and households, such as whether to use their funds to invest in new equipment for factories or to save their money in a bank.

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Under "Rates & Bonds," you can access information on key interest rates, U.S. Treasuries, Government bonds, and municipal bonds.

Before we can go on with the study of money, banking, and financial markets, we must understand exactly what the phrase *interest rates* means. In this chapter, we see that a concept known as the *yield to maturity* is the most accurate measure of interest rates; the yield to maturity is what economists mean when they use the term *interest rate*. We discuss how the yield to maturity is measured and examine alternative (but less accurate) ways in which interest rates are quoted. We'll also see that a bond's interest rate does not necessarily indicate how good an investment the bond is because what it earns (its rate of return) does not necessarily equal its interest rate. Finally, we explore the distinction between real interest rates, which are adjusted for inflation, and nominal interest rates, which are not.

Although learning definitions is not always the most exciting of pursuits, it is important to read carefully and understand the concepts presented in this chapter. Not only are they continually used throughout the remainder of this text, but a firm grasp of these terms will give you a clearer understanding of the role that interest rates play in your life as well as in the general economy.

Measuring Interest Rates

Different debt instruments have very different streams of payment with very different timing. Thus we first need to understand how we can compare the value of one kind of debt instrument with another before we see how interest rates are measured. To do this, we make use of the concept of *present value*.

Present Value

The concept of **present value** (or **present discounted value**) is based on the commonsense notion that a dollar paid to you one year from now is less valuable to you than a dollar paid to you today: This notion is true because you can deposit a dollar in a savings account that earns interest and have more than a dollar in one year. Economists use a more formal definition, as explained in this section.

Let's look at the simplest kind of debt instrument, which we will call a **simple loan**. In this loan, the lender provides the borrower with an amount of funds (called the *principal*) that must be repaid to the lender at the *maturity date*, along with an additional payment for the interest. For example, if you made your friend, Jane, a simple loan of \$100 for one year, you would require her to repay the principal of \$100 in one year's time along with an additional payment for interest; say, \$10. In the case of a simple loan like this one, the interest payment divided by the amount of the loan is a natural and sensible way to measure the interest rate. This measure of the so-called *simple interest rate*, i, is:

$$i = \frac{\$10}{\$100} = 0.10 = 10\%$$

If you make this \$100 loan, at the end of the year you would have \$110, which can be rewritten as:

$$\$100 \times (1 + 0.10) = \$110$$

If you then lent out the \$110, at the end of the second year you would have:

$$\$110 \times (1 + 0.10) = \$121$$

or, equivalently,

$$\$100 \times (1 + 0.10) \times (1 + 0.10) = \$100 \times (1 + 0.10)^2 = \$121$$

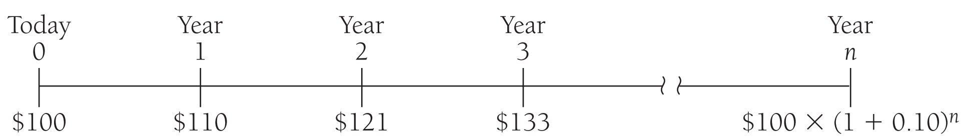
Continuing with the loan again, you would have at the end of the third year:

$$\$121 \times (1 + 0.10) = \$100 \times (1 + 0.10)^3 = \$133$$

Generalizing, we can see that at the end of n years, your \$100 would turn into:

$$\$100 \times (1 + i)^n$$

The amounts you would have at the end of each year by making the \$100 loan today can be seen in the following timeline:



This timeline immediately tells you that you are just as happy having \$100 today as having \$110 a year from now (of course, as long as you are sure that Jane will pay you back). Or that you are just as happy having \$100 today as having \$121 two years from now, or \$133 three years from now or $\$100 \times (1 + 0.10)^n$, n years from now. The timeline tells us that we can also work backward from future amounts to the present: for example, $\$133 = \$100 \times (1 + 0.10)^3$ three years from now is worth \$100 today, so that:

$$\$100 = \frac{\$133}{(1 + 0.10)^3}$$

The process of calculating today's value of dollars received in the future, as we have done above, is called *discounting the future*. We can generalize this process by writing today's (present) value of $100 as *PV*, the future value of $133 as *FV*, and replacing 0.10 (the 10% interest rate) by *i*. This leads to the following formula:

$$PV = \frac{FV}{(1 + i)^n} \tag{1}$$

Intuitively, what Equation 1 tells us is that if you are promised $1 for certain ten years from now, this dollar would not be as valuable to you as $1 is today because if you had the $1 today, you could invest it and end up with more than $1 in ten years.

The concept of present value is extremely useful, because it allows us to figure out today's value (price) of a credit market instrument at a given simple interest rate *i* by just adding up the individual present values of all the future payments received. This information allows us to compare the value of two instruments with very different timing of their payments.

As an example of how the present value concept can be used, let's assume that you just hit the $20 million jackpot in the New York State Lottery, which promises you a payment of $1 million for the next twenty years. You are clearly excited, but have you really won $20 million? No, not in the present value sense. In today's dollars, that $20 million is worth a lot less. If we assume an interest rate of 10% as in the earlier examples, the first payment of $1 million is clearly worth $1 million today, but the next payment next year is only worth $1 million/(1 + 0.10) = $909,090, a lot less than $1 million. The following year the payment is worth $1 million/$(1 + 0.10)^2$ = $826,446 in today's dollars, and so on. When you add all these up, they come to $9.4 million. You are still pretty excited (who wouldn't be?), but because you understand the concept of present value, you recognize that you are the victim of false advertising. You didn't really win $20 million, but instead won less than half as much.

Four Types of Credit Market Instruments

In terms of the timing of their payments, there are four basic types of credit market instruments.

1. A simple loan, which we have already discussed, in which the lender provides the borrower with an amount of funds, which must be repaid to the lender at the maturity date along with an additional payment for the interest. Many money market instruments are of this type: for example, commercial loans to businesses.

2. A **fixed-payment loan** (which is also called a **fully amortized loan**) in which the lender provides the borrower with an amount of funds, which must be repaid by making the same payment every period (such as a month), consisting of part of the principal and interest for a set number of years. For example, if you borrowed $1,000, a fixed-payment loan might require you to pay $126 every year for 25 years. Installment loans (such as auto loans) and mortgages are frequently of the fixed-payment type.

3. A **coupon bond** pays the owner of the bond a fixed interest payment (coupon payment) every year until the maturity date, when a specified final amount (**face value** or **par value**) is repaid. The coupon payment is so named because the bondholder used to obtain payment by clipping a coupon off the bond and sending it to the bond issuer, who then sent the payment to the holder. Nowadays, it is no longer necessary to send in coupons to receive these payments. A coupon bond with $1,000 face value, for example, might pay you a coupon payment of $100 per year for ten years, and at the maturity date repay you the face value amount of $1,000. (The face value of a bond is usually in $1,000 increments.)

A coupon bond is identified by three pieces of information. First is the corporation or government agency that issues the bond. Second is the maturity date of the bond. Third is the bond's **coupon rate**, the dollar amount of the yearly coupon payment expressed as a percentage of the face value of the bond. In our example, the coupon bond has a yearly coupon payment of \$100 and a face value of \$1,000. The coupon rate is then \$100/\$1,000 = 0.10, or 10%. Capital market instruments such as U.S. Treasury bonds and notes and corporate bonds are examples of coupon bonds.

4. A **discount bond** (also called a **zero-coupon bond**) is bought at a price below its face value (at a discount), and the face value is repaid at the maturity date. Unlike a coupon bond, a discount bond does not make any interest payments; it just pays off the face value. For example, a discount bond with a face value of \$1,000 might be bought for \$900; in a year's time the owner would be repaid the face value of \$1,000. U.S. Treasury bills, U.S. savings bonds, and long-term zero-coupon bonds are examples of discount bonds.

These four types of instruments require payments at different times: Simple loans and discount bonds make payment only at their maturity dates, whereas fixed-payment loans and coupon bonds have payments periodically until maturity. How would you decide which of these instruments provides you with more income? They all seem so different because they make payments at different times. To solve this problem, we use the concept of present value, explained earlier, to provide us with a procedure for measuring interest rates on these different types of instruments.

Yield to Maturity

Of the several common ways of calculating interest rates, the most important is the **yield to maturity**, the interest rate that equates the present value of payments received from a debt instrument with its value today.[1] Because the concept behind the calculation of the yield to maturity makes good economic sense, economists consider it the most accurate measure of interest rates.

To understand the yield to maturity better, we now look at how it is calculated for the four types of credit market instruments.

Simple Loan. Using the concept of present value, the yield to maturity on a simple loan is easy to calculate. For the one-year loan we discussed, today's value is \$100, and the payments in one year's time would be \$110 (the repayment of \$100 plus the interest payment of \$10). We can use this information to solve for the yield to maturity i by recognizing that the present value of the future payments must equal today's value of a loan. Making today's value of the loan (\$100) equal to the present value of the \$110 payment in a year (using Equation 1) gives us:

$$\$100 = \frac{\$110}{1 + i}$$

Solving for i,

$$i = \frac{\$110 - \$100}{\$100} = \frac{\$10}{\$100} = 0.10 = 10\%$$

This calculation of the yield to maturity should look familiar, because it equals the interest payment of \$10 divided by the loan amount of \$100; that is, it equals the simple interest rate on the loan. An important point to recognize is that *for simple loans, the simple interest rate equals the yield to maturity*. Hence the same term i is used to denote both the yield to maturity and the simple interest rate.

[1]In other contexts, it is also called the *internal rate of return*.

Study Guide

The key to understanding the calculation of the yield to maturity is equating today's value of the debt instrument with the present value of all of its future payments. The best way to learn this principle is to apply it to other specific examples of the four types of credit market instruments in addition to those we discuss here. See if you can develop the equations that would allow you to solve for the yield to maturity in each case.

Fixed-Payment Loan. Recall that this type of loan has the same payment every period throughout the life of the loan. On a fixed-rate mortgage, for example, the borrower makes the same payment to the bank every month until the maturity date, when the loan will be completely paid off. To calculate the yield to maturity for a fixed-payment loan, we follow the same strategy we used for the simple loan—we equate today's value of the loan with its present value. Because the fixed-payment loan involves more than one payment, the present value of the fixed-payment loan is calculated as the sum of the present values of all payments (using Equation 1).

In the case of our earlier example, the loan is \$1,000 and the yearly payment is \$126 for the next 25 years. The present value is calculated as follows: At the end of one year, there is a \$126 payment with a *PV* of $\$126/(1 + i)$; at the end of two years, there is another \$126 payment with a *PV* of $\$126/(1 + i)^2$; and so on until at the end of the twenty-fifth year, the last payment of \$126 with a *PV* of $\$126/(1 + i)^{25}$ is made. Making today's value of the loan (\$1,000) equal to the sum of the present values of all the yearly payments gives us:

$$\$1{,}000 = \frac{\$126}{1+i} + \frac{\$126}{(1+i)^2} + \frac{\$126}{(1+i)^3} + \cdots + \frac{\$126}{(1+i)^{25}}$$

More generally, for any fixed-payment loan,

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \cdots + \frac{FP}{(1+i)^n} \tag{2}$$

where LV = loan value
FP = fixed yearly payment
n = number of years until maturity

For a fixed-payment loan amount, the fixed yearly payment and the number of years until maturity are known quantities, and only the yield to maturity is not. So we can solve this equation for the yield to maturity i. Because this calculation is not easy, many pocket calculators have programs that allow you to find i given the loan's numbers for LV, FP, and n. For example, in the case of the 25-year loan with yearly payments of \$126, the yield to maturity that solves Equation 2 is 12%. Real estate brokers always have a pocket calculator that can solve such equations so that they can immediately tell the prospective house buyer exactly what the yearly (or monthly) payments will be if the house purchase is financed by taking out a mortgage.[2]

Coupon Bond. To calculate the yield to maturity for a coupon bond, follow the same strategy used for the fixed-payment loan: Equate today's value of the bond with its present value. Because coupon bonds also have more than one payment, the present

[2]The calculation with a pocket calculator programmed for this purpose requires simply that you enter the value of the loan LV, the number of years to maturity n, and the interest rate i and then run the program.

value of the bond is calculated as the sum of the present values of all the coupon payments plus the present value of the final payment of the face value of the bond.

The present value of a $1,000-face-value bond with ten years to maturity and yearly coupon payments of $100 (a 10% coupon rate) can be calculated as follows: At the end of one year, there is a $100 coupon payment with a *PV* of $\$100/(1 + i)$; at the end of the second year, there is another $100 coupon payment with a *PV* of $\$100/(1 + i)^2$; and so on until at maturity, there is a $100 coupon payment with a *PV* of $\$100/(1 + i)^{10}$ plus the repayment of the $1,000 face value with a *PV* of $\$1{,}000/(1 + i)^{10}$. Setting today's value of the bond (its current price, denoted by *P*) equal to the sum of the present values of all the payments for this bond gives:

$$P = \frac{\$100}{1 + i} + \frac{\$100}{(1 + i)^2} + \frac{\$100}{(1 + i)^3} + \cdots + \frac{\$100}{(1 + i)^{10}} + \frac{\$1{,}000}{(1 + i)^{10}}$$

More generally, for any coupon bond,[3]

$$P = \frac{C}{1 + i} + \frac{C}{(1 + i)^2} + \frac{C}{(1 + i)^3} + \cdots + \frac{C}{(1 + i)^n} + \frac{F}{(1 + i)^n} \qquad (3)$$

where
P = price of coupon bond
C = yearly coupon payment
F = face value of the bond
n = years to maturity date

In Equation 3, the coupon payment, the face value, the years to maturity, and the price of the bond are known quantities, and only the yield to maturity is not. Hence we can solve this equation for the yield to maturity *i*. Just as in the case of the fixed-payment loan, this calculation is not easy, so business-oriented pocket calculators have built-in programs that solve this equation for you.[4]

Let's look at some examples of the solution for the yield to maturity on our 10%-coupon-rate bond that matures in ten years. If the purchase price of the bond is $1,000, then either using a pocket calculator with the built-in program or looking at a bond table, we will find that the yield to maturity is 10 percent. If the price is $900, we find that the yield to maturity is 11.75%. Table 1 shows the yields to maturity calculated for several bond prices.

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

[3]Most coupon bonds actually make coupon payments on a semiannual basis rather than once a year as assumed here. The effect on the calculations is only very slight and will be ignored here.

[4]The calculation of a bond's yield to maturity with the programmed pocket calculator requires simply that you enter the amount of the yearly coupon payment *C*, the face value *F*, the number of years to maturity *n*, and the price of the bond *P* and then run the program.

Three interesting facts are illustrated by Table 1:

1. When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.
2. The price of a coupon bond and the yield to maturity are negatively related; that is, as the yield to maturity rises, the price of the bond falls. As the yield to maturity falls, the price of the bond rises.
3. The yield to maturity is greater than the coupon rate when the bond price is below its face value.

These three facts are true for any coupon bond and are really not surprising if you think about the reasoning behind the calculation of the yield to maturity. When you put $1,000 in a bank account with an interest rate of 10%, you can take out $100 every year and you will be left with the $1,000 at the end of ten years. This is similar to buying the $1,000 bond with a 10% coupon rate analyzed in Table 1, which pays a $100 coupon payment every year and then repays $1,000 at the end of ten years. If the bond is purchased at the par value of $1,000, its yield to maturity must equal 10%, which is also equal to the coupon rate of 10%. The same reasoning applied to any coupon bond demonstrates that if the coupon bond is purchased at its par value, the yield to maturity and the coupon rate must be equal.

It is straightforward to show that the bond price and the yield to maturity are negatively related. As i, the yield to maturity, rises, all denominators in the bond price formula must necessarily rise. Hence a rise in the interest rate as measured by the yield to maturity means that the price of the bond must fall. Another way to explain why the bond price falls when the interest rises is that a higher interest rate implies that the future coupon payments and final payment are worth less when discounted back to the present; hence the price of the bond must be lower.

There is one special case of a coupon bond that is worth discussing because its yield to maturity is particularly easy to calculate. This bond is called a **consol** or a **perpetuity**; it is a perpetual bond with no maturity date and no repayment of principal that makes fixed coupon payments of $$C$ forever. Consols were first sold by the British Treasury during the Napoleonic Wars and are still traded today; they are quite rare, however, in American capital markets. The formula in Equation 3 for the price of the consol P simplifies to the following:[5]

$$P = \frac{C}{i} \tag{4}$$

[5]The bond price formula for a consol is:

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots$$

which can be written as:

$$P = C\,(x + x^2 + x^3 + \cdots)$$

in which $x = 1/(1 + i)$. The formula for an infinite sum is:

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} \quad \text{for} \quad x < 1$$

and so:

$$P = C\left(\frac{1}{1-x} - 1\right) = C\left[\frac{1}{1 - 1/(1+i)} - 1\right]$$

which by suitable algebraic manipulation becomes:

$$P = C\left(\frac{1+i}{i} - \frac{i}{i}\right) = \frac{C}{i}$$

where P = price of the consol
C = yearly payment

One nice feature of consols is that you can immediately see that as i goes up, the price of the bond falls. For example, if a consol pays \$100 per year forever and the interest rate is 10%, its price will be \$1,000 = \$100/0.10. If the interest rate rises to 20%, its price will fall to \$500 = \$100/0.20. We can also rewrite this formula as

$$i = \frac{C}{P} \tag{5}$$

We see then that it is also easy to calculate the yield to maturity for the consol (despite the fact that it never matures). For example, with a consol that pays \$100 yearly and has a price of \$2,000, the yield to maturity is easily calculated to be 5% (= \$100/\$2,000).

Discount Bond. The yield-to-maturity calculation for a discount bond is similar to that for the simple loan. Let us consider a discount bond such as a one-year U.S. Treasury bill, which pays off a face value of \$1,000 in one year's time. If the current purchase price of this bill is \$900, then equating this price to the present value of the \$1,000 received in one year, using Equation 1, gives:

$$\$900 = \frac{\$1{,}000}{1 + i}$$

and solving for i,

$$\begin{aligned}(1 + i) \times \$900 &= \$1{,}000\\ \$900 + \$900i &= \$1{,}000\\ \$900i &= \$1{,}000 - \$900\\ i &= \frac{\$1{,}000 - \$900}{\$900} = 0.111 = 11.1\%\end{aligned}$$

More generally, for any one-year discount bond, the yield to maturity can be written as:

$$i = \frac{F - P}{P} \tag{6}$$

where F = face value of the discount bond
P = current price of the discount bond

In other words, the yield to maturity equals the increase in price over the year $F - P$ divided by the initial price P. In normal circumstances, investors earn positive returns from holding these securities and so they sell at a discount, meaning that the current price of the bond is below the face value. Therefore, $F - P$ should be positive, and the yield to maturity should be positive as well. However, this is not always the case, as recent extraordinary events in Japan indicate (see Box 1).

An important feature of this equation is that it indicates that for a discount bond, the yield to maturity is negatively related to the current bond price. This is the same conclusion that we reached for a coupon bond. For example, Equation 6 shows that a rise in the bond price from \$900 to \$950 means that the bond will have a smaller

Box 1: Global

Negative T-Bill Rates? Japan Shows the Way

We normally assume that interest rates must always be positive. Negative interest rates would imply that you are willing to pay more for a bond today than you will receive for it in the future (as our formula for yield to maturity on a discount bond demonstrates). Negative interest rates therefore seem like an impossibility because you would do better by holding cash that has the same value in the future as it does today.

The Japanese have demonstrated that this reasoning is not quite correct. In November 1998, interest rates on Japanese six-month Treasury bills became negative, yielding an interest rate of –0.004%, with investors paying more for the bills than their face value. This is an extremely unusual event—no other country in the world has seen negative interest rates during the last fifty years. How could this happen?

As we will see in Chapter 5, the weakness of the Japanese economy and a negative inflation rate drove Japanese interest rates to low levels, but these two factors can't explain the negative rates. The answer is that large investors found it more convenient to hold these six-month bills as a store of value rather than holding cash because the bills are denominated in larger amounts and can be stored electronically. For that reason, some investors were willing to hold them, despite their negative rates, even though in monetary terms the investors would be better off holding cash. Clearly, the convenience of T-bills goes only so far, and thus their interest rates can go only a little bit below zero.

increase in its price at maturity, and the yield to maturity falls from 11.1 to 5.3%. Similarly, a fall in the yield to maturity means that the price of the discount bond has risen.

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A review of the key financial concepts: time value of money, annuities, perpetuities, and so on.

Summary. The concept of present value tells you that a dollar in the future is not as valuable to you as a dollar today because you can earn interest on this dollar. Specifically, a dollar received n years from now is worth only $\$1/(1 + i)^n$ today. The present value of a set of future payments on a debt instrument equals the sum of the present values of each of the future payments. The yield to maturity for an instrument is the interest rate that equates the present value of the future payments on that instrument to its value today. Because the procedure for calculating the yield to maturity is based on sound economic principles, this is the measure that economists think most accurately describes the interest rate.

Our calculations of the yield to maturity for a variety of bonds reveal the important fact that ***current bond prices and interest rates are negatively related: When the interest rate rises, the price of the bond falls, and vice versa***.

Other Measures of Interest Rates

The yield to maturity is the most accurate measure of interest rates; this is what economists mean when they use the term *interest rate*. Unless otherwise specified, the terms *interest rate* and *yield to maturity* are used synonymously in this book. However, because the yield to maturity is sometimes difficult to calculate, other, less accurate measures of interest rates have come into common use in bond markets. You will frequently encounter two of these measures—the *current yield* and the *yield on a discount basis*—when reading the newspaper, and it is important for you to understand what they mean and how they differ from the more accurate measure of interest rates, the yield to maturity.

Current Yield

The **current yield** is an approximation of the yield to maturity on coupon bonds that is often reported, because in contrast to the yield to maturity, it is easily calculated. It is defined as the yearly coupon payment divided by the price of the security,

$$i_c = \frac{C}{P} \tag{7}$$

where i_c = current yield
P = price of the coupon bond
C = yearly coupon payment

This formula is identical to the formula in Equation 5, which describes the calculation of the yield to maturity for a consol. Hence, for a consol, the current yield is an exact measure of the yield to maturity. When a coupon bond has a long term to maturity (say, 20 years or more), it is very much like a consol, which pays coupon payments forever. Thus you would expect the current yield to be a rather close approximation of the yield to maturity for a long-term coupon bond, and you can safely use the current-yield calculation instead of calculating the yield to maturity with a financial calculator. However, as the time to maturity of the coupon bond shortens (say, it becomes less than five years), it behaves less and less like a consol and so the approximation afforded by the current yield becomes worse and worse.

We have also seen that when the bond price equals the par value of the bond, the yield to maturity is equal to the coupon rate (the coupon payment divided by the par value of the bond). Because the current yield equals the coupon payment divided by the bond price, the current yield is also equal to the coupon rate when the bond price is at par. This logic leads us to the conclusion that when the bond price is at par, the current yield equals the yield to maturity. This means that the closer the bond price is to the bond's par value, the better the current yield will approximate the yield to maturity.

The current yield is negatively related to the price of the bond. In the case of our 10%-coupon-rate bond, when the price rises from $1,000 to $1,100, the current yield falls from 10% (= $100/$1,000) to 9.09% (= $100/$1,100). As Table 1 indicates, the yield to maturity is also negatively related to the price of the bond; when the price rises from $1,000 to $1,100, the yield to maturity falls from 10 to 8.48%. In this we see an important fact: The current yield and the yield to maturity always move together; a rise in the current yield always signals that the yield to maturity has also risen.

The general characteristics of the current yield (the yearly coupon payment divided by the bond price) can be summarized as follows: The current yield better approximates the yield to maturity when the bond's price is nearer to the bond's par value and the maturity of the bond is longer. It becomes a worse approximation when the bond's price is further from the bond's par value and the bond's maturity is shorter. Regardless of whether the current yield is a good approximation of the yield to maturity, a change in the current yield *always* signals a change in the same direction of the yield to maturity.

Yield on a Discount Basis

Before the advent of calculators and computers, dealers in U.S. Treasury bills found it difficult to calculate interest rates as a yield to maturity. Instead, they quoted the interest rate on bills as a **yield on a discount basis** (or **discount yield**), and they still do so today. Formally, the yield on a discount basis is defined by the following formula:

$$i_{db} = \frac{F - P}{F} \times \frac{360}{\textit{days to maturity}} \tag{8}$$

where i_{db} = yield on a discount basis
F = face value of the discount bond
P = purchase price of the discount bond

This method for calculating interest rates has two peculiarities. First, it uses the percentage gain on the face value of the bill $(F - P)/F$ rather than the percentage gain on the purchase price of the bill $(F - P)/P$ used in calculating the yield to maturity. Second, it puts the yield on an annual basis by considering the year to be 360 days long rather than 365 days.

Because of these peculiarities, the discount yield understates the interest rate on bills as measured by the yield to maturity. On our one-year bill, which is selling for \$900 and has a face value of \$1,000, the yield on a discount basis would be as follows:

$$i_{db} = \frac{\$1{,}000 - \$900}{\$1{,}000} \times \frac{360}{365} = 0.099 = 9.9\%$$

whereas the yield to maturity for this bill, which we calculated before, is 11.1%. The discount yield understates the yield to maturity by a factor of over 10%. A little more than 1% ([365 − 360]/360 = 0.014 = 1.4%) can be attributed to the understatement of the length of the year: When the bill has one year to maturity, the second term on the right-hand side of the formula is 360/365 = 0.986 rather than 1.0, as it should be.

The more serious source of the understatement, however, is the use of the percentage gain on the face value rather than on the purchase price. Because, by definition, the purchase price of a discount bond is always less than the face value, the percentage gain on the face value is necessarily smaller than the percentage gain on the purchase price. The greater the difference between the purchase price and the face value of the discount bond, the more the discount yield understates the yield to maturity. Because the difference between the purchase price and the face value gets larger as maturity gets longer, we can draw the following conclusion about the relationship of the yield on a discount basis to the yield to maturity: The yield on a discount basis always understates the yield to maturity, and this understatement becomes more severe the longer the maturity of the discount bond.

Another important feature of the discount yield is that, like the yield to maturity, it is negatively related to the price of the bond. For example, when the price of the bond rises from \$900 to \$950, the formula indicates that the yield on a discount basis declines from 9.9 to 4.9%. At the same time, the yield to maturity declines from 11.1 to 5.3%. Here we see another important factor about the relationship of yield on a discount basis to yield to maturity: They always move together. That is, a rise in the discount yield always means that the yield to maturity has risen, and a decline in the discount yield means that the yield to maturity has declined as well.

The characteristics of the yield on a discount basis can be summarized as follows: Yield on a discount basis understates the more accurate measure of the interest rate, the yield to maturity; and the longer the maturity of the discount bond, the greater this understatement becomes. Even though the discount yield is a somewhat misleading measure of the interest rates, a change in the discount yield always indicates a change in the same direction for the yield to maturity.

Application

Reading the *Wall Street Journal*: The Bond Page

Now that we understand the different interest-rate definitions, let's apply our knowledge and take a look at what kind of information appears on the bond page of a typical newspaper, in this case the *Wall Street Journal.* The "Following the Financial News" box contains the *Journal*'s listing for three different types of bonds on Wednesday, January 23, 2003. Panel (a) contains the information on U.S. Treasury bonds and notes. Both are coupon bonds, the only difference being their time to maturity from when they were originally issued: Notes have a time to maturity of less than ten years; bonds have a time to maturity of more than ten years.

The information found in the "Rate" and "Maturity" columns identifies the bond by coupon rate and maturity date. For example, T-bond 1 has a coupon rate of 4.75%, indicating that it pays out \$47.50 per year on a \$1,000-face-value bond and matures in January 2003. In bond market parlance, it is referred to as the Treasury's $4\frac{3}{4}$s of 2003. The next three columns tell us about the bond's price. By convention, all prices in the bond market are quoted per \$100 of face value. Furthermore, the numbers after the colon represent thirty-seconds ($^x/_{32}$, or 32nds). In the case of T-bond 1, the first price of 100:02 represents $100\frac{2}{32} = 100.0625$, or an actual price of \$1000.62 for a \$1,000-face-value bond. The bid price tells you what price you will receive if you sell the bond, and the asked price tells you what you must pay for the bond. (You might want to think of the bid price as the "wholesale" price and the asked price as the "retail" price.) The "Chg." column indicates how much the bid price has changed in 32nds (in this case, no change) from the previous trading day.

Notice that for all the bonds and notes, the asked price is more than the bid price. Can you guess why this is so? The difference between the two (the *spread*) provides the bond dealer who trades these securities with a profit. For T-bond 1, the dealer who buys it at $100\frac{2}{32}$, and sells it for $100\frac{3}{32}$, makes a profit of $\frac{1}{32}$. This profit is what enables the dealer to make a living and provide the service of allowing you to buy and sell bonds at will.

The "Ask Yld." column provides the yield to maturity, which is 0.43% for T-bond 1. It is calculated with the method described earlier in this chapter using the asked price as the price of the bond. The asked price is used in the calculation because the yield to maturity is most relevant to a person who is going to buy and hold the security and thus earn the yield. The person selling the security is not going to be holding it and hence is less concerned with the yield.

The figure for the current yield is not usually included in the newspaper's quotations for Treasury securities, but it has been added in panel (a) to give you some real-world examples of how well the current yield approximates

Following the Financial News

Bond Prices and Interest Rates

Bond prices and interest rates are published daily. In the *Wall Street Journal*, they can be found in the "NYSE/AMEX Bonds" and "Treasury/Agency Issues" section of the paper. Three basic formats for quoting bond prices and yields are illustrated here.

TREASURY BONDS, NOTES AND BILLS

January 22, 2003

Representative Over-the-Counter quotation based on transactions of $1 million or more.

Treasury bond, note and bill quotes are as of mid-afternoon. Colons in bid-and-asked quotes represent 32nds; 101:01 means 101 1/32. Net changes in 32nds. n-Treasury note. i-Inflation-Indexed issue. Treasury bill quotes in hundredths, quoted on terms of a rate of discount. Days to maturity calculated from settlement date. All yields are to maturity and based on the asked quote. Latest 13-week and 26-week bills are boldfaced. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par. *When issued.

Source: eSpeed/Cantor Fitzgerald

U.S. Treasury strips as of 3 p.m. Eastern time, also based on transactions of $1 million or more. Colons in bid and asked quotes represent 32nds; 99:01 means 99 1/32. Net changes in 32nds. Yields calculated on the asked quotation. ci-stripped coupon interest. bp-Treasury bond, stripped principal. np-Treasury note, stripped principal. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

Source: Bear, Stearns & Co. via Street Software Technology, Inc.

(a) Treasury bonds and notes

GOVT. BONDS & NOTES

	Rate	Maturity Mo/Yr	Bid	Asked	Chg.	Ask Yld.	
T-bond 1	4.750	Jan 03n	100:02	100:03	. . .	0.43	Current Yield = 4.75%
	5.500	Jan 03n	100:02	100:03	–1	0.46	
	5.750	Aug 03n	102:17	102:18	. . .	0.16	
T-bond 2	11.125	Aug 03	105:16	105:17	–1	1.22	Current Yield = 10.55%
T-bond 3	5.250	Feb 29	103:17	103:18	23	5.00	Current Yield = 5.07%
	3.875	Apr 29i	122:03	122:04	2	2.69	
	6.125	Aug 29	116:10	116:11	24	5.00	
T-bond 4	5.375	Feb 31	107:27	107:28	24	4.86	Current Yield = 4.98%

(b) Treasury bills

TREASURY BILLS

Maturity	Days to Mat.	Bid	Asked	Chg.	Ask Yld.	Maturity	Days to Mat.	Bid	Asked	Chg.	Ask Yld.
Jan 30 03	7	1.15	1.14	-0.01	1.16	May 01 03	98	1.14	1.13	-0.02	1.15
Feb 06 03	14	1.14	1.13	-0.01	1.15	May 08 03	105	1.14	1.13	-0.03	1.15
Feb 13 03	21	1.14	1.13	-0.01	1.15	May 15 03	112	1.15	1.14	-0.02	1.16
Feb 20 03	28	1.14	1.13	. . .	1.15	May 22 03	119	1.15	1.14	-0.02	1.16
Feb 27 03	35	1.13	1.12	-0.01	1.14	May 29 03	126	1.15	1.14	-0.01	1.16
Mar 06 03	42	1.13	1.12	. . .	1.14	Jun 05 03	133	1.15	1.14	-0.02	1.16
Mar 13 03	49	1.13	1.12	-0.01	1.14	Jun 12 03	140	1.16	1.15	-0.01	1.17
Mar 20 03	56	1.12	1.11	-0.01	1.13	Jun 19 03	147	1.15	1.14	-0.02	1.16
Mar 27 03	63	1.13	1.12	-0.01	1.14	Jun 26 03	154	1.15	1.14	-0.01	1.16
Apr 03 03	70	1.13	1.12	-0.01	1.14	Jul 03 03	161	1.15	1.14	-0.02	1.16
Apr 10 03	77	1.12	1.11	-0.03	1.13	Jul 10 03	168	1.16	1.15	-0.02	1.17
Apr 17 03	84	1.14	1.13	-0.01	1.15	Jul 17 03	175	1.16	1.15	-0.03	1.17
Apr 24 03	91	1.15	1.14	. . .	1.16	Jul 24 03	182	1.17	1.16	. . .	1.18

(c) New York Stock Exchange bonds

NEW YORK BONDS
CORPORATION BONDS

	Bonds	Cur Yld	Vol	Close	Net Chg.	
Bond 1	AT&T $5\frac{5}{8}$04	5.5	238	101.63	. . .	Yield to Maturity = 3.68%
	AT&T $6\frac{3}{8}$04	6.2	60	102.63	–0.13	
	AT&T $7\frac{1}{2}$04	7.2	101	103.63	–0.13	
	AT&T $8\frac{1}{8}$24	8.0	109	101	0.38	
	ATT 8.35s25	8.3	60	101	0.50	
	AT&T $6\frac{1}{2}$29	7.5	190	87.25	0.13	
Bond 2	AT&T $8\frac{5}{8}$31	8.4	138	102.75	0.88	Yield to Maturity = 8.40%

Source: Wall Street Journal, Thursday, January 23, 2003, p. C11.

the yield to maturity. Our previous discussion provided us with some rules for deciding when the current yield is likely to be a good approximation and when it is not.

T-bonds 3 and 4 mature in around 30 years, meaning that their characteristics are like those of a consol. The current yields should then be a good approximation of the yields to maturity, and they are: The current yields are within two-tenths of a percentage point of the values for the yields to maturity. This approximation is reasonable even for T-bond 4, which has a price about 7% above its face value.

Now let's take a look at T-bonds 1 and 2, which have a much shorter time to maturity. The price of T-bond 1 differs by less than 1% from the par value, and look how poor an approximation the current yield is for the yield to maturity; it overstates the yield to maturity by more than 4 percentage points. The approximation for T-bond 2 is even worse, with the overstatement over 9 percentage points. This bears out what we learned earlier about the current yield: It can be a very misleading guide to the value of the yield to maturity for a short-term bond if the bond price is not extremely close to par.

Two other categories of bonds are reported much like the Treasury bonds and notes in the newspaper. Government agency and miscellaneous securities include securities issued by U.S. government agencies such as the Government National Mortgage Association, which makes loans to savings and loan institutions, and international agencies such as the World Bank. Tax-exempt bonds are the other category reported in a manner similar to panel (a), except that yield-to-maturity calculations are not usually provided. Tax-exempt bonds include bonds issued by local government and public authorities whose interest payments are exempt from federal income taxes.

Panel (b) quotes yields on U.S. Treasury bills, which, as we have seen, are discount bonds. Since there is no coupon, these securities are identified solely by their maturity dates, which you can see in the first column. The next column, "Days to Mat.," provides the number of days to maturity of the bill. Dealers in these markets always refer to prices by quoting the yield on a discount basis. The "Bid" column gives the discount yield for people selling the bills to dealers, and the "Asked" column gives the discount yield for people buying the bills from dealers. As with bonds and notes, the dealers' profits are made by the asked price being higher than the bid price, leading to the asked discount yield being lower than the bid discount yield.

The "Chg." column indicates how much the asked discount yield changed from the previous day. When financial analysts talk about changes in the yield, they frequently describe the changes in terms of **basis points**, which are hundredths of a percentage point. For example, a financial analyst would describe the -0.01 change in the asked discount yield for the February 13, 2003, T-bill by saying that it had fallen by 1 basis point.

As we learned earlier, the yield on a discount basis understates the yield to maturity, which is reported in the column of panel (b) headed "Ask Yld." This is evident from a comparison of the "Ask Yld." and "Asked" columns. As we would also expect from our discussion of the calculation of yields on a discount basis, the understatement grows as the maturity of the bill lengthens.

Panel (c) has quotations for corporate bonds traded on the New York Stock Exchange. Corporate bonds traded on the American Stock Exchange are reported in like manner. The first column identifies the bond by indicating the corporation that issued it. The bonds we are looking at have all been issued by American Telephone and Telegraph (AT&T). The next column tells the coupon rate and the maturity date ($5\frac{5}{8}$ and 2004 for Bond 1). The "Cur. Yld." column reports the current yield (5.5), and "Vol." gives the volume of trading in that bond (238 bonds of $1,000 face value traded that day). The "Close" price is the last traded price that day per $100 of face value. The price of 101.63 represents $1016.30 for a $1,000-face-value bond. The "Net Chg." is the change in the closing price from the previous trading day.

The yield to maturity is also given for two bonds. This information is not usually provided in the newspaper, but it is included here because it shows how misleading the current yield can be for a bond with a short maturity such as the $5\frac{5}{8}$s, of 2004. The current yield of 5.5% is a misleading measure of the interest rate because the yield to maturity is actually 3.68 percent. By contrast, for the $8\frac{5}{8}$s, of 2031, with nearly 30 years to maturity, the current yield and the yield to maturity are exactly equal.

The Distinction Between Interest Rates and Returns

Many people think that the interest rate on a bond tells them all they need to know about how well off they are as a result of owning it. If Irving the Investor thinks he is better off when he owns a long-term bond yielding a 10% interest rate and the interest rate rises to 20%, he will have a rude awakening: As we will shortly see, if he has to sell the bond, Irving has lost his shirt! How well a person does by holding a bond or any other security over a particular time period is accurately measured by the **return**, or, in more precise terminology, the **rate of return**. For any security, the rate of return is defined as the payments to the owner plus the change in its value, expressed as a fraction of its purchase price. To make this definition clearer, let us see what the return would look like for a $1,000-face-value coupon bond with a coupon rate of 10% that is bought for $1,000, held for one year, and then sold for $1,200. The payments to the owner are the yearly coupon payments of $100, and the change in its value is $1,200 − $1,000 = $200. Adding these together and expressing them as a fraction of the purchase price of $1,000 gives us the one-year holding-period return for this bond:

$$\frac{\$100 + \$200}{\$1,000} = \frac{\$300}{\$1,000} = 0.30 = 30\%$$

You may have noticed something quite surprising about the return that we have just calculated: It equals 30%, yet as Table 1 indicates, initially the yield to maturity was only 10 percent. This demonstrates that ***the return on a bond will not necessarily equal the interest rate on that bond***. We now see that the distinction between interest rate and return can be important, although for many securities the two may be closely related.

Study Guide

The concept of *return* discussed here is extremely important because it is used continually throughout the book. Make sure that you understand how a return is calculated and why it can differ from the interest rate. This understanding will make the material presented later in the book easier to follow.

More generally, the return on a bond held from time t to time $t + 1$ can be written as:

$$RET = \frac{C + P_{t+1} - P_t}{P_t} \tag{9}$$

where
RET = return from holding the bond from time t to time $t + 1$
P_t = price of the bond at time t
P_{t+1} = price of the bond at time $t + 1$
C = coupon payment

A convenient way to rewrite the return formula in Equation 9 is to recognize that it can be split into two separate terms:

$$RET = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

The first term is the current yield i_c (the coupon payment over the purchase price):

$$\frac{C}{P_t} = i_c$$

The second term is the **rate of capital gain**, or the change in the bond's price relative to the initial purchase price:

$$\frac{P_{t+1} - P_t}{P_t} = g$$

where g = rate of capital gain. Equation 9 can then be rewritten as:

$$RET = i_c + g \tag{10}$$

which shows that the return on a bond is the current yield i_c plus the rate of capital gain g. This rewritten formula illustrates the point we just discovered. Even for a bond for which the current yield i_c is an accurate measure of the yield to maturity, the return can differ substantially from the interest rate. Returns will differ from the interest rate, especially if there are sizable fluctuations in the price of the bond that produce substantial capital gains or losses.

To explore this point even further, let's look at what happens to the returns on bonds of different maturities when interest rates rise. Table 2 calculates the one-year return on several 10%-coupon-rate bonds all purchased at par when interest rates on

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

all these bonds rise from 10 to 20%. Several key findings in this table are generally true of all bonds:

- The only bond whose return equals the initial yield to maturity is one whose time to maturity is the same as the holding period (see the last bond in Table 2).
- A rise in interest rates is associated with a fall in bond prices, resulting in capital losses on bonds whose terms to maturity are longer than the holding period.
- The more distant a bond's maturity, the greater the size of the percentage price change associated with an interest-rate change.
- The more distant a bond's maturity, the lower the rate of return that occurs as a result of the increase in the interest rate.
- Even though a bond has a substantial initial interest rate, its return can turn out to be negative if interest rates rise.

At first it frequently puzzles students (as it puzzles poor Irving the Investor) that a rise in interest rates can mean that a bond has been a poor investment. The trick to understanding this is to recognize that a rise in the interest rate means that the price of a bond has fallen. A rise in interest rates therefore means that a capital loss has occurred, and if this loss is large enough, the bond can be a poor investment indeed. For example, we see in Table 2 that the bond that has 30 years to maturity when purchased has a capital loss of 49.7% when the interest rate rises from 10 to 20%. This loss is so large that it exceeds the current yield of 10%, resulting in a negative return (loss) of −39.7%. If Irving does not sell the bond, his capital loss is often referred to as a "paper loss." This is a loss nonetheless because if he had not bought this bond and had instead put his money in the bank, he would now be able to buy more bonds at their lower price than he presently owns.

Maturity and the Volatility of Bond Returns: Interest-Rate Risk

The finding that the prices of longer-maturity bonds respond more dramatically to changes in interest rates helps explain an important fact about the behavior of bond markets: ***Prices and returns for long-term bonds are more volatile than those for shorter-term bonds***. Price changes of +20% and −20% within a year, with corresponding variations in returns, are common for bonds more than 20 years away from maturity.

We now see that changes in interest rates make investments in long-term bonds quite risky. Indeed, the riskiness of an asset's return that results from interest-rate changes is so important that it has been given a special name, **interest-rate risk**.[6] Dealing with interest-rate risk is a major concern of managers of financial institutions and investors, as we will see in later chapters (see also Box 2).

Although long-term debt instruments have substantial interest-rate risk, short-term debt instruments do not. Indeed, bonds with a maturity that is as short as the holding period have no interest-rate risk.[7] We see this for the coupon bond at the bottom of Table 2, which has no uncertainty about the rate of return because it equals the yield to maturity, which is known at the time the bond is purchased. The key to understanding why there is no interest-rate risk for *any* bond whose time to maturity matches the holding period is to recognize that (in this case) the price at the end of the holding period is already fixed at the face value. The change in interest rates can then have no effect on the price at the end of the holding period for these bonds, and the return will therefore be equal to the yield to maturity known at the time the bond is purchased.[8]

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[6]Interest-rate risk can be quantitatively measured using the concept of *duration*. This concept and how it is calculated is discussed in an appendix to this chapter, which can be found on this book's web site at www.aw.com/mishkin.

[7]The statement that there is no interest-rate risk for any bond whose time to maturity matches the holding period is literally true only for discount bonds and zero-coupon bonds that make no intermediate cash payments before the holding period is over. A coupon bond that makes an intermediate cash payment before the holding period is over requires that this payment be reinvested. Because the interest rate at which this payment can be reinvested is uncertain, there is some uncertainty about the return on this coupon bond even when the time to maturity equals the holding period. However, the riskiness of the return on a coupon bond from reinvesting the coupon payments is typically quite small, and so the basic point that a coupon bond with a time to maturity equaling the holding period has very little risk still holds true.

[8]In the text, we are assuming that all holding periods are short and equal to the maturity on short-term bonds and are thus not subject to interest-rate risk. However, if an investor's holding period is longer than the term to maturity of the bond, the investor is exposed to a type of interest-rate risk called *reinvestment risk*. Reinvestment risk occurs because the proceeds from the short-term bond need to be reinvested at a future interest rate that is uncertain.

To understand reinvestment risk, suppose that Irving the Investor has a holding period of two years and decides to purchase a $1,000 one-year bond at face value and will then purchase another one at the end of the first year. If the initial interest rate is 10%, Irving will have $1,100 at the end of the year. If the interest rate rises to 20%, as in Table 2, Irving will find that buying $1,100 worth of another one-year bond will leave him at the end of the second year with $1,100 × (1 + 0.20) = $1,320. Thus Irving's two-year return will be ($1,320 − $1,000)/1,000 = 0.32 = 32%, which equals 14.9% at an annual rate. In this case, Irving has earned more by buying the one-year bonds than if he had initially purchased the two-year bond with an interest rate of 10%. Thus when Irving has a holding period that is longer than the term to maturity of the bonds he purchases, he benefits from a rise in interest rates. Conversely, if interest rates fall to 5%, Irving will have only $1,155 at the end of two years: $1,100 × (1 + 0.05). Thus his two-year return will be ($1,155 − $1,000)/1,000 = 0.155 = 15.5%, which is 7.2 percent at an annual rate. With a holding period greater than the term to maturity of the bond, Irving now loses from a fall in interest rates.

We have thus seen that when the holding period is longer than the term to maturity of a bond, the return is uncertain because the future interest rate when reinvestment occurs is also uncertain—in short, there is reinvestment risk. We also see that if the holding period is longer than the term to maturity of the bond, the investor benefits from a rise in interest rates and is hurt by a fall in interest rates.

Box 2

Helping Investors to Select Desired Interest-Rate Risk

Because many investors want to know how much interest-rate risk they are exposed to, some mutual fund companies try to educate investors about the perils of interest-rate risk, as well as to offer investment alternatives that match their investors' preferences.

Vanguard Group, for example, offers eight separate high-grade bond mutual funds. In its prospectus, Vanguard separates the funds by the average maturity of the bonds they hold and demonstrates the effect of interest-rate changes by computing the percentage change in bond value resulting from a 1% increase and decrease in interest rates. Three of the bond funds invest in bonds with average maturities of one to three years, which Vanguard rates as having the lowest interest-rate risk. Three other funds hold bonds with average maturities of five to ten years, which Vanguard rates as having medium interest-rate risk. Two funds hold long-term bonds with maturities of 15 to 30 years, which Vanguard rates as having high interest-rate risk.

By providing this information, Vanguard hopes to increase its market share in the sales of bond funds. Not surprisingly, Vanguard is one of the most successful mutual fund companies in the business.

Summary

The return on a bond, which tells you how good an investment it has been over the holding period, is equal to the yield to maturity in only one special case: when the holding period and the maturity of the bond are identical. Bonds whose term to maturity is longer than the holding period are subject to interest-rate risk: Changes in interest rates lead to capital gains and losses that produce substantial differences between the return and the yield to maturity known at the time the bond is purchased. Interest-rate risk is especially important for long-term bonds, where the capital gains and losses can be substantial. This is why long-term bonds are not considered to be safe assets with a sure return over short holding periods.

The Distinction Between Real and Nominal Interest Rates

www.martincapital.com/charts.htm

Go to charts of real versus nominal rates to view 30 years of nominal interest rates compared to real rates for the 30-year T-bond and 90-day T-bill.

So far in our discussion of interest rates, we have ignored the effects of inflation on the cost of borrowing. What we have up to now been calling the interest rate makes no allowance for inflation, and it is more precisely referred to as the **nominal interest rate**, which is distinguished from the **real interest rate**, the interest rate that is adjusted by subtracting expected changes in the price level (inflation) so that it more accurately reflects the true cost of borrowing.[9] The real interest rate is more accurately defined by the *Fisher equation*, named for Irving Fisher, one of the great monetary economists of the

[9]The real interest rate defined in the text is more precisely referred to as the *ex ante real interest rate* because it is adjusted for *expected* changes in the price level. This is the real interest rate that is most important to economic decisions, and typically it is what economists mean when they make reference to the "real" interest rate. The interest rate that is adjusted for *actual* changes in the price level is called the *ex post real interest rate.* It describes how well a lender has done in real terms *after the fact.*

twentieth century. The Fisher equation states that the nominal interest rate i equals the real interest rate i_r plus the expected rate of inflation π^e:[10]

$$i = i_r + \pi^e \tag{11}$$

Rearranging terms, we find that the real interest rate equals the nominal interest rate minus the expected inflation rate:

$$i_r = i - \pi^e \tag{12}$$

To see why this definition makes sense, let us first consider a situation in which you have made a one-year simple loan with a 5% interest rate ($i = 5\%$) and you expect the price level to rise by 3% over the course of the year ($\pi^e = 3\%$). As a result of making the loan, at the end of the year you will have 2% more in **real terms**, that is, in terms of real goods and services you can buy. In this case, the interest rate you have earned in terms of real goods and services is 2%; that is,

$$i_r = 5\% - 3\% = 2\%$$

as indicated by the Fisher definition.

Now what if the interest rate rises to 8%, but you expect the inflation rate to be 10% over the course of the year? Although you will have 8% more dollars at the end of the year, you will be paying 10% more for goods; the result is that you will be able to buy 2% fewer goods at the end of the year and you are 2% worse off *in real terms*. This is also exactly what the Fisher definition tells us, because:

$$i_r = 8\% - 10\% = -2\%$$

As a lender, you are clearly less eager to make a loan in this case, because in terms of real goods and services you have actually earned a negative interest rate of 2%. By contrast, as the borrower, you fare quite well because at the end of the year, the amounts you will have to pay back will be worth 2% less in terms of goods and services—you as the borrower will be ahead by 2% in real terms. ***When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.***

A similar distinction can be made between nominal returns and real returns. Nominal returns, which do not allow for inflation, are what we have been referring to as simply "returns." When inflation is subtracted from a nominal return, we have the real return, which indicates the amount of extra goods and services that can be purchased as a result of holding the security.

The distinction between real and nominal interest rates is important because the real interest rate, which reflects the real cost of borrowing, is likely to be a better indicator of the incentives to borrow and lend. It appears to be a better guide to how peo-

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[10]A more precise formulation of the Fisher equation is:

$$i = i_r + \pi^e + (i_r \times \pi^e)$$

because:

$$1 + i = (1 + i_r)(1 + \pi^e) = 1 + i_r + \pi^e + (i_r \times \pi^e)$$

and subtracting 1 from both sides gives us the first equation. For small values of i_r and π^e, the term $i_r \times \pi^e$ is so small that we ignore it, as in the text.

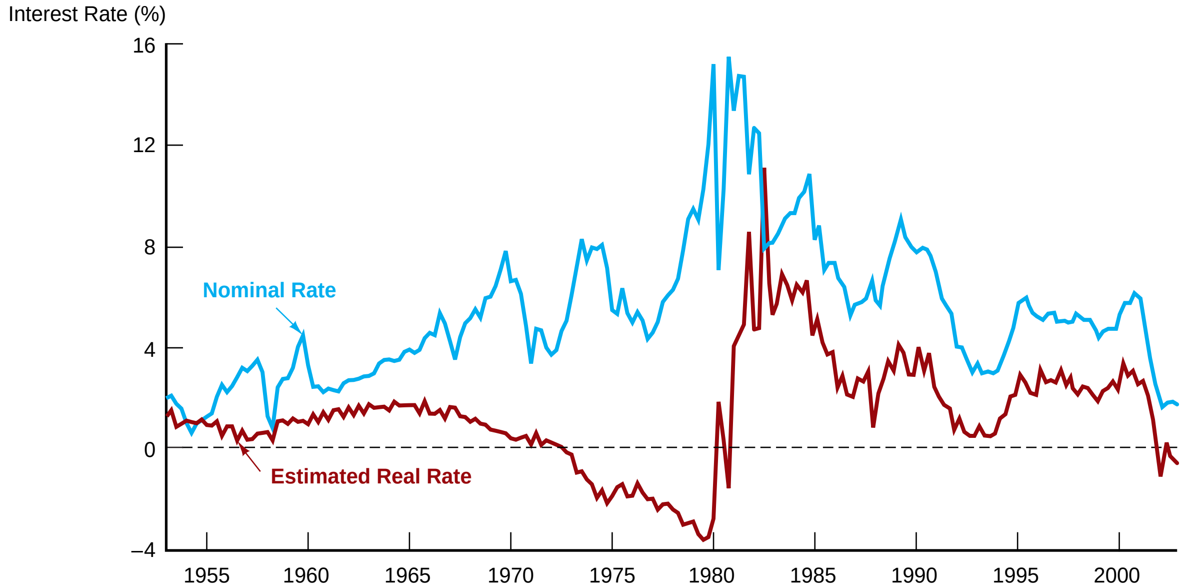

FIGURE 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2002

Sources: Nominal rates from www.federalreserve.gov/releases/H15. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy* 15 (1981): 151–200. This procedure involves estimating expected inflation as a function of past interest rates, inflation, and time trends and then subtracting the expected inflation measure from the nominal interest rate.

ple will be affected by what is happening in credit markets. Figure 1, which presents estimates from 1953 to 2002 of the real and nominal interest rates on three-month U.S. Treasury bills, shows us that nominal and real rates often do not move together. (This is also true for nominal and real interest rates in the rest of the world.) In particular, when nominal rates in the United States were high in the 1970s, real rates were actually extremely low—often negative. By the standard of nominal interest rates, you would have thought that credit market conditions were tight in this period, because it was expensive to borrow. However, the estimates of the real rates indicate that you would have been mistaken. In real terms, the cost of borrowing was actually quite low.[11]

[11]Because most interest income in the United States is subject to federal income taxes, the true earnings in real terms from holding a debt instrument are not reflected by the real interest rate defined by the Fisher equation but rather by the *after-tax real interest rate,* which equals the nominal interest rate *after income tax payments have been subtracted,* minus the expected inflation rate. For a person facing a 30% tax rate, the after-tax interest rate earned on a bond yielding 10% is only 7% because 30% of the interest income must be paid to the Internal Revenue Service. Thus the after-tax real interest rate on this bond when expected inflation is 5% equals 2% (= 7% − 5%). More generally, the after-tax real interest rate can be expressed as:

$$i(1 - \tau) - \pi^e$$

where τ = the income tax rate.

This formula for the after-tax real interest rate also provides a better measure of the effective cost of borrowing for many corporations and homeowners in the United States because in calculating income taxes, they can deduct

Box 3

With TIPS, Real Interest Rates Have Become Observable in the United States

When the U.S. Treasury decided to issue TIPS (Treasury Inflation Protection Securities), in January 1997, a version of indexed Treasury coupon bonds, it was somewhat late in the game. Other countries such as the United Kingdom, Canada, Australia, and Sweden had already beaten the United States to the punch. (In September 1998, the U.S. Treasury also began issuing the Series I savings bond, which provides inflation protection for small investors.)

These indexed securities have successfully acquired a niche in the bond market, enabling governments to raise more funds. In addition, because their interest and principal payments are adjusted for changes in the price level, the interest rate on these bonds provides a direct measure of a real interest rate.

These indexed bonds are very useful to policymakers, especially monetary policymakers, because by subtracting their interest rate from a nominal interest rate on a nonindexed bond, they generate more insight into expected inflation, a valuable piece of information. For example, on January 22, 2003, the interest rate on the ten-year Treasury bond was 3.84%, while that on the ten-year TIPS was 2.19%. Thus, the implied expected inflation rate for the next ten years, derived from the difference between these two rates, was 1.65%. The private sector finds the information provided by TIPS very useful: Many commercial and investment banks routinely publish the expected U.S. inflation rates derived from these bonds.

Until recently, real interest rates in the United States were not observable; only nominal rates were reported. This all changed when, in January 1997, the U.S. Treasury began to issue **indexed bonds**, whose interest and principal payments are adjusted for changes in the price level (see Box 3).

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interest payments on loans from their income. Thus if you face a 30% tax rate and take out a mortgage loan with a 10% interest rate, you are able to deduct the 10% interest payment and thus lower your taxes by 30% of this amount. Your after-tax nominal cost of borrowing is then 7% (10% minus 30% of the 10% interest payment), and when the expected inflation rate is 5%, the effective cost of borrowing in real terms is again 2% (= 7% − 5%).

As the example (and the formula) indicates, after-tax real interest rates are always below the real interest rate defined by the Fisher equation. For a further discussion of measures of after-tax real interest rates, see Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy* 15 (1981): 151–200.

Summary

1. The yield to maturity, which is the measure that most accurately reflects the interest rate, is the interest rate that equates the present value of future payments of a debt instrument with its value today. Application of this principle reveals that bond prices and interest rates are negatively related: When the interest rate rises, the price of the bond must fall, and vice versa.
2. Two less accurate measures of interest rates are commonly used to quote interest rates on coupon and discount bonds. The current yield, which equals the coupon payment divided by the price of a coupon bond, is a less accurate measure of the yield to maturity the shorter the maturity of the bond and the greater the gap between the price and the par value. The yield on a discount basis (also called the discount yield) understates the yield to maturity on a discount bond, and the understatement worsens with the distance from maturity of the discount security. Even though these

measures are misleading guides to the size of the interest rate, a change in them always signals a change in the same direction for the yield to maturity.

3. The return on a security, which tells you how well you have done by holding this security over a stated period of time, can differ substantially from the interest rate as measured by the yield to maturity. Long-term bond prices have substantial fluctuations when interest rates change and thus bear interest-rate risk. The resulting capital gains and losses can be large, which is why long-term bonds are not considered to be safe assets with a sure return.

4. The real interest rate is defined as the nominal interest rate minus the expected rate of inflation. It is a better measure of the incentives to borrow and lend than the nominal interest rate, and it is a more accurate indicator of the tightness of credit market conditions than the nominal interest rate.

Key Terms

basis point, p. 74
consol or perpetuity, p. 67
coupon bond, p. 63
coupon rate, p. 64
current yield, p. 70
discount bond (zero-coupon bond), p. 64
face value (par value), p. 63
fixed-payment loan (fully amortized loan), p. 63
indexed bond, p. 82
interest-rate risk, p. 78
nominal interest rate, p. 79
present discounted value, p. 61
present value, p. 61
rate of capital gain, p. 76
real interest rate, p. 79
real terms, p. 80
return (rate of return), p. 75
simple loan, p. 62
yield on a discount basis (discount yield), p. 71
yield to maturity, p. 64

Questions and Problems

Questions marked with an asterisk are answered at the end of the book in an appendix, "Answers to Selected Questions and Problems."

*1. Would a dollar tomorrow be worth more to you today when the interest rate is 20% or when it is 10%?

2. You have just won $20 million in the state lottery, which promises to pay you $1 million (tax free) every year for the next 20 years. Have you really won $20 million?

*3. If the interest rate is 10%, what is the present value of a security that pays you $1,100 next year, $1,210 the year after, and $1,331 the year after that?

4. If the security in Problem 3 sold for $3,500, is the yield to maturity greater or less than 10%? Why?

*5. Write down the formula that is used to calculate the yield to maturity on a 20-year 10% coupon bond with $1,000 face value that sells for $2,000.

6. What is the yield to maturity on a $1,000-face-value discount bond maturing in one year that sells for $800?

*7. What is the yield to maturity on a simple loan for $1 million that requires a repayment of $2 million in five years' time?

8. To pay for college, you have just taken out a $1,000 government loan that makes you pay $126 per year for 25 years. However, you don't have to start making these payments until you graduate from college two years from now. Why is the yield to maturity necessarily less than 12%, the yield to maturity on a normal $1,000 fixed-payment loan in which you pay $126 per year for 25 years?

*9. Which $1,000 bond has the higher yield to maturity, a 20-year bond selling for $800 with a current yield of 15% or a one-year bond selling for $800 with a current yield of 5%?

10. Pick five U.S. Treasury bonds from the bond page of the newspaper, and calculate the current yield. Note when the current yield is a good approximation of the yield to maturity.

*11. You are offered two bonds, a one-year U.S. Treasury bond with a yield to maturity of 9% and a one-year U.S. Treasury bill with a yield on a discount basis of 8.9%. Which would you rather own?

12. If there is a decline in interest rates, which would you rather be holding, long-term bonds or short-term bonds? Why? Which type of bond has the greater interest-rate risk?

*13. Francine the Financial Adviser has just given you the following advice: "Long-term bonds are a great investment because their interest rate is over 20%." Is Francine necessarily right?

14. If mortgage rates rise from 5% to 10% but the expected rate of increase in housing prices rises from 2% to 9%, are people more or less likely to buy houses?

*15. Interest rates were lower in the mid-1980s than they were in the late 1970s, yet many economists have commented that real interest rates were actually much higher in the mid-1980s than in the late 1970s. Does this make sense? Do you think that these economists are right?

Web Exercises

1. Investigate the data available from the Federal Reserve at www.federalreserve.gov/releases/. Answer the following questions:
   a. What is the difference in the interest rates on commercial paper for financial firms when compared to nonfinancial firms?
   b. What was the interest rate on the one-month Eurodollar at the end of 2002?
   c. What is the most recent interest rate report for the 30-year Treasury note?

2. Figure 1 in the text shows the estimated real and nominal rates for three-month treasury bills. Go to www.martincapital.com/charts.htm and click on "interest rates and yields," then on "real interest rates."
   a. Compare the three-month real rate to the long-term real rate. Which is greater?
   b. Compare the short-term nominal rate to the long-term nominal rate. Which appears most volatile?

3. In this chapter we have discussed long-term bonds as if there were only one type, coupon bonds. In fact there are also long-term discount bonds. A discount bond is sold at a low price and the whole return comes in the form of a price appreciation. You can easily compute the current price of a discount bond using the financial calculator at http://app.ny.frb.org/sbr/.

   To compute the redemption values for savings bonds, fill in the information at the site and click on the Compute Values button. A maximum of five years of data will be displayed for each computation.